Money, Tobin's q and Investment Dynamics in the Small Open Economy

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Money, Tobin's q and Investment Dynamics in the Small Open Economy

Geld, Tobin's q und die Investitionsdynamik in einer kleinen offenen Volkswirtschaft

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1. Introduction

In a small open economy the steady state capital stock is determined by the country's investment opportunities and the world real interest rate. This can be shown in MacDougall's framework, where investment opportunities are represented by the downward-sloping schedule of the marginal product of capital[1]). With homogenous and internationally tradable claims to capital goods (equities) the domestic capital stock is at its steady state level when the marginal product of capital equals the world real interest rate. In this setting the domestic capital stock is independent of domestic wealth. The difference between domestic holdings of equities and the total number of domestic equities is absorbed by foreign investors.

Niehans (1984) shows how the capital stock reaches the described steady state value when a country opens its borders to international trade in equities. In this dynamic adjustment process the price of capital goods measured in consumption goods (the value of the equity) plays an important role. A small country that opens up to the international capital market will experience a jump in the price of its equities if the domestic marginal product of capital exceeds the world level. It is through this equity-price adjustment that the international yield difference is eliminated. The increase in the market price of capital goods relative to the replacement costs of capital (the Tobin-q) then calls forth investment. The process of capital accumulation is terminated when the domestic marginal product of capital has reached the world level.

The q-investment theory was originally developed to clarify the transmission of monetary impulses to the real side of the economy. In Tobin's (1969) famous article an increase in the supply of money leads to an excess demand for interest-bearing assets

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[1]) See *MacDougall* (1960) or *Ruffin* (1984).

and drives up the prices of these assets. This revaluation of asset prices equilibrates money and asset markets and stimulates output by triggering investment. The present article links the money-induced investment with the international determination of the equilibrium capital stock. Contrary to Niehans' (1987) two-country case I study a model where monetary policy has no permanent effect on the capital stock.

The small economy considered here operates under flexible exchange rates and with perfect foresight. The analysis focuses on the interplay of the interest rate, the Tobin-q and investment. The model does not determine consumption and income. A model covering consumption and income would be much more complex, especially due to the wealth effects and terms-of-trade effects that would have to be considered[2]). Since income and consumption are not covered it is not possible to trace the reaction of savings[3]). Hence, the present analysis does not tell to what extent investment is financed by domestic or foreign funds. Nevertheless, as I shall point out at the end of the paper, the results obtained here shed light on the reaction of the current account to a money supply shock.

2. *The Model*

We consider a world with no growth and no secular inflation. Physical capital goods cannot be shifted between countries[4]). However, claims to capital goods are tradable, and domestic and foreign equities are perfect substitutes. Money is the only asset besides equities. This means e.g. that no foreign moneys and thus no currency substitution are considered. The central bank controls the nominal money supply through transfers to the public. The public controls real money balances by changing the price level. We assume that the price level does not adjust immediately to a change in the money supply. Therefore, the domestic interest rate plays an equilibrating role till prices reach their new equilibrium. This liquidity effect is compatible with international yield equality, when the expected change in the exchange rate matches the interest rate differential. The exchange rate behaviour implied by our model therefore features the Dornbusch overshooting characteristic: i.e. an increase in the money supply leads to an immediate depreciation of the exchange rate[5]). Since in this paper the real exchange rate is assumed to have no influence on the interest rate and capital formation it is not dwelled on any further. The liquidity effect is modeled by solving the money market equilibrium condition for the interest rate[6]):

$$i = j - b(m - \bar{m}) \qquad b > 0\,. \tag{1}$$

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[2]) Covering these aspects would add several differential equations which would make intuitive interpretation and the extracting of general economic propositions impossible. *Lipton* and *Sachs* (1983) e.g. have to resort to simulation analysis because their model consists of 6 differential equations. Their work comprises a two-country model, but they do not introduce money.

[3]) See *Dornbusch* and *Fischer* (1980) for an analysis focusing on the savings dynamics.

[4]) There are, of course, mobile capital goods such as airplaines. However, as *Niehans* (1987) points out, the immobility assumption is probably more realistic overall since most capital goods, such as buildings and roads, cannot be shifted, once built.

[5]) See *Dornbusch* (1976) and *Niehans* (1984, chapter 11).

[6]) See *Blanchard* (1981).

The variables introduced are, respectively, i the domestic nominal interest rate, j the foreign interest rate (abroad the nominal and the real interest rate are identical and equal the marginal product of capital), m the log of deflated money balances and $\overline{m}$ the long-run equilibrium level of real balances. The system is driven by an unexpected step change in the nominal money supply. The price level is assumed to adjust gradually to the disturbance. In this article the stickiness of the price level is accepted as a fact of economic life and is not elaborated on any further[7]). I use the simplest method to model both the stickiness property of the price level and its long-run (quantity theoretical) property of adjusting proportionally to changes in the money stock by writing a differential equation that rules the behaviour of real balances after a step change in the money supply (a dot denotes the time derivative of a variable):

$$\dot{m} = -\lambda(m - \overline{m}) \qquad \lambda > 0 . \tag{2}$$

The change over time of real balances described by equation (2) is only due to the adjustment of the price level since the nominal money stock remains constant after the step change in its level. Hence, $\dot{m}$ is identical to $-\dot{p}$ (the inflation rate multiplied by -1). The real rate of interest i^r is defined as:

$$i^r = i - \dot{p} . \tag{3}$$

The domestic marginal product of capital (r, the rental) is assumed to be a decreasing function of the capital stock (K)[8]):

$$r = j + v(K - \overline{K}) \qquad v < 0 . \tag{4}$$

$\overline{K}$ denotes the steady state equilibrium capital stock. If $K = \overline{K}$, the domestic marginal product of capital equals the foreign interest rate. The change of the capital stock over time ($\dot{K}$, investment) is a function of the difference between the market value of capital goods (q, the equity price) and the replacement costs of capital ($\overline{q} = 1$):

$$\dot{K} = h(q - \overline{q}) \qquad h > 0 . \tag{5}$$

In the present model the real interest rate is synonymous with the return on holding one equity. This return is composed of the rental and the capital gain ($\dot{q}$, the change of the equity price):

$$i^r = \frac{r + \dot{q}}{q} . \tag{6}$$

This link between the real interest rate, the capital rental and the equity price is the same as in Blanchard's model of a closed economy[9]).

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[7]) Modern research on price level inertia focuses on the costs firms incur when changing their prices, the so-called menu costs. See e.g. *Mankiw* (1990) for recent developments in this field.

[8]) A linear relationship between r and K simplifies matters considerably.

[9]) See *Blanchard* (1981).

3. *Solution*

The model presented can be reduced to a system of three linear differential equations (7–9). Equation 8 is the result of inserting equation 3 into equation 6 (with $\dot{p} = -\dot{m}$) and linearization around $\overline{m}$ and $\overline{q}$. Equation 9 is reached by differentiating equation 4 with respect to time and inserting equation 5.

$$\dot{m} = -\lambda(m - \overline{m}) \tag{7}$$

$$\dot{q} = -\overline{q}(b + \lambda)(m - \overline{m}) + j(q - \overline{q}) - 1(r - \bar{r}) \tag{8}$$

$$\dot{r} = vh(q - \overline{q}) \tag{9}$$

The system is a saddle-point equilibrium, which is a common feature of similar perfect foresight models[10]). The equity-price is the only variable in this system that can move endogenously at infinite speed. Following the literature, we therefore assume that q is bid immediately to the level that sets the system on the only stable trajectory[11]). The experiment conducted is an unexpected step increase in the money supply (dm). The corresponding algebraic solutions for m, q and r can be found in the appendix. The solutions for the remaining variables are found by inserting the expressions for m, q and r in the respective equations.

4. *Results*

Figure 1 shows how the variables of the model evolve. The path of real balances decreases over time because the price level adjusts to the initial increase in the money supply[12]). The nominal and the real interest rate fall instantaneously due to the excess liquidity in the economy. Thereafter, they rise steadily towards their common equilibrium level. During the stage of adjustment (with transitory inflation) i^r is lower than i. The counterpart of the immediate decline in the interest rate is a steep rise in the equity price. Both the higher level of q and its expected decline bring about the lower return on equities (the interest rate) at which people willingly hold the extra real balances. Over time, the equity price sinks below $\overline{q}$ and again rises to this steady-state value.

Investment mimics the behaviour of q. In the first stage of the adjustment process (when $q > \overline{q}$) capital accumulation is positive. In the second stage (when $q < \overline{q}$) capital accumulation is negative. We can think of the second stage as one in which not all the worn-out capital goods are replaced. The first stage is shorter and shows a stronger absolute reaction of investment than the second one. However, aggregated over time, investment and disinvestment are equal in magnitude. Thus, the two successive stages ensure the long-run neutrality of the capital stock with respect to money[13]). The marginal product of capital sinks in the first stage when the capital stock grows and

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[10]) See *Gray* and *Turnovsky* (1979).

[11]) See *Blanchard* (1981).

[12]) The price level rises from its initial equilibrium level $\overline{p}_0$ to $\overline{p}_1$. The price level is the only variable that does not return to its initial value.

[13]) During the adjustment process the capital stock and production capacity are elevated.

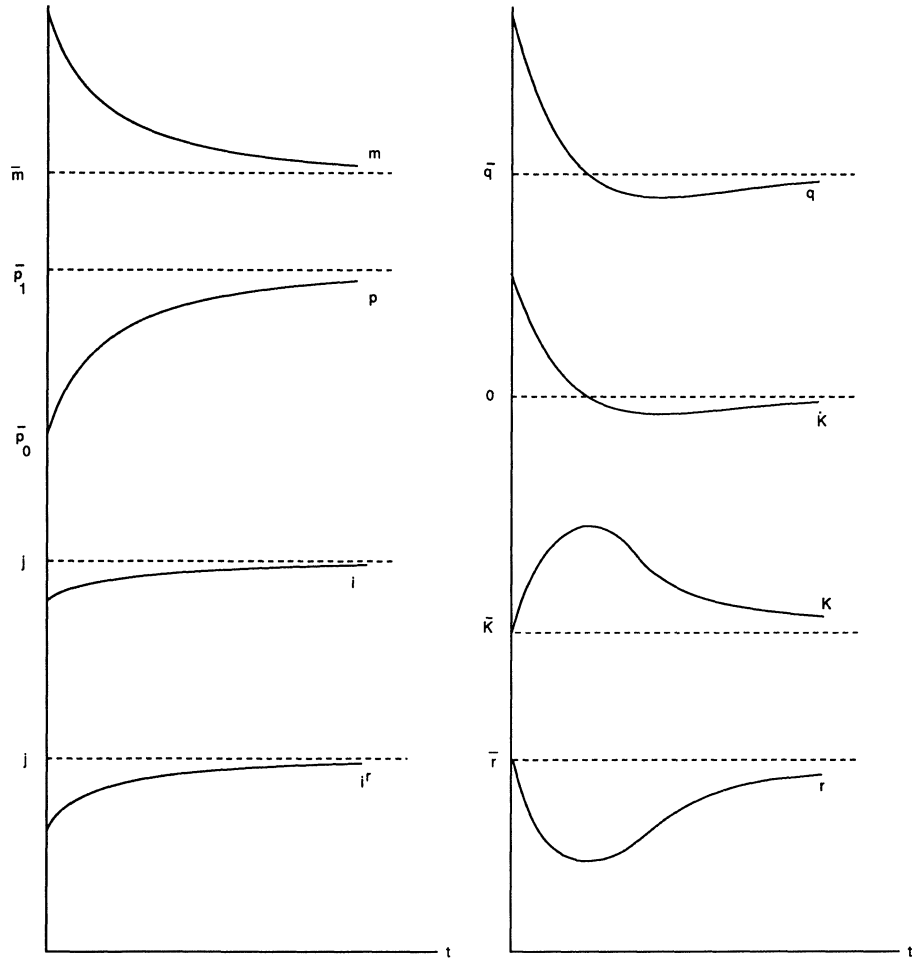

Fig. 1: Reaction of the Variables to a Monetary Expansion.

rises in the second stage when the capital stock sinks back towards its steady-state equilibrium.

Our analysis also bears on the question whether an increase in the money supply is followed by a surplus or a deficit in the current account. The standard Mundell-Fleming model predicts a current account surplus for this case[14]). Johnson (1972) argued that the investment induced by the expansionary monetary policy can outweigh the counteracting effects and lead to a current account deficit. In this article no attempt is made to cover the effects of the monetary expansion on savings. Still, the strong first-stage investment implied by our model supports Johnson's view. The disinvestment of the second stage, however, is conducive to a current account surplus and is

[14]) See *Mundell* (1962, 1963) and *Fleming* (1962).

therefore more in line with the Mundell-Fleming view. As a whole, the two-stage pattern of investment analyzed in this article influences the current account in the direction of the J-effect known from the elasticity approach to the balance of payments[15]).

Appendix

The system of the differential equations (7–9) can be written in matrix form:

$$\begin{bmatrix} \dot{m} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\lambda & 0 & 0 \\ -\bar{q}(b+\lambda) & j & -1 \\ 0 & vh & 0 \end{bmatrix} \begin{bmatrix} m-\bar{m} \\ q-\bar{q} \\ r-\bar{r} \end{bmatrix}.$$

Solving the eigenvalue problem, we reach the characteristic equation in the roots (ζ):

$$(-\lambda-\zeta)(-\zeta j+\zeta^2+vh)=0\,.$$

There are three roots: two of them (ζ_1 and ζ_2) are negative and one (ζ_3) is positive. The system is therefore a saddle-point equilibrium.

$$\zeta_1 = -\lambda < 0$$

$$\zeta_2 = \frac{j-\sqrt{j^2-4vh}}{2} < 0$$

$$\zeta_3 = -\zeta_2 > 0$$

The system only converges to its steady state when the factor of integration of the positive root is zero. The solutions for m, q and r are then derived after finding the characteristic vectors:

$$m = a_1 e^{\zeta_1 t} + \bar{m}$$

$$q = a_1 \frac{\bar{q}(b+\lambda)\zeta_1}{j\zeta_1-\zeta_1^2-vh} e^{\zeta_1 t} + a_2 e^{\zeta_2 t} + \bar{q}$$

$$r = a_1 \frac{vh\bar{q}(b+\lambda)}{j\zeta_1-\zeta_1^2-vh} e^{\zeta_1 t} + a_2 \frac{vh}{\zeta_2} e^{\zeta_2 t} + \bar{r}\,.$$

The variables a_1 and a_2 are determined by initial conditions for m and r:

$$a_1 = dm$$

$$a_2 = -\frac{\bar{q}(b+\lambda)\zeta_2}{j\zeta_1-\zeta_1^2-vh}\,dm.$$

The instantaneous reaction of q is definitely positive. As q can only have one interior maximum or minimum (for $t>0$) and since the area between q and $\bar{q}$ (positive for $q>\bar{q}$ and negative for $q<\bar{q}$) has to sum to zero (to ascertain that K goes to $\bar{K}$ as t approaches ∞) the characteristics of the q-path are clear-cut.

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[15]) See *Niehans* (1984).

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Summary

This paper investigates the dynamic response of the Tobin-q and investment to an unexpected increase in the money supply. The problem is analyzed in a simple dynamic model of a stationary small open economy. We find a first, relatively short, stage with strong investment followed by a second stage with moderate disinvestment. This pattern of investment influences the current account in the direction of the well-known J-effect.

Zusammenfassung

In diesem Artikel wird die dynamische Reaktion des Tobin-q und der gesamtwirtschaftlichen Investitionen auf einen monetären Impuls untersucht. Dazu wird ein einfaches Modell einer stationären kleinen offenen Volkswirtschaft entwickelt. Es zeigt sich, daß die Investitionen während einer ersten, relativ kurzen, Phase stark positiv sind und daß danach über längere Zeit desinvestiert wird. Dieses Zeitprofil der volkswirtschaftlichen Kapitalakkumulation beeinflußt die Leistungsbilanz in Richtung des vom Elastizitätsansatz her bekannten J-Effektes.

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