Price and Output Effects of Heterogeneous Expectations

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1. INTRODUCTION

This article takes up several questions relating to the heterogeneity of expectations and mean bias in expectations. One common proposition designed to motivate the use of the rational expectations hypothesis is that individual errors in expectations do not matter for economic outcomes as long as the expectations of the population are on average unbiased. As DAVID BEGG, the author of one popular textbook on the subject, puts it: «... one can appeal to the Law of Large Numbers which argues that individual idiosyncrasies are likely to cancel out in the aggregate, leaving average behavior rather closer to the implications of the theory».1 This suggests that we rely on the predictions of models built on the assumption that all individuals can be represented by a single rational individual. The study of the limitations of this claim is an important research topic.2 Recently, this topic has been explored most actively in the finance literature. DE LONG, SUMMERS, SHLEIFER and WALDMANN (1990) show that when the population consists of both rational and irrational investors – the latter are called noise traders – the equilibrium asset price will deviate from the rational expectations value. KAZEMI (1991) studies the effect of dispersion of beliefs on asset prices in a noisy rational expectations framework. He finds that increasing dispersion of beliefs can either raise or lower prices. In his framework the dispersion of beliefs is endogenous and the direction of the aforementioned effect is largely determined by what causes an increase in the dispersion of beliefs. Macroeconomic effects of expectations heterogeneity have been studied by TOWNSEND (1983) and HALITWANGER and WALDMAN (1989).

The present article formulates three simple models in which heterogeneity of expectations influences land rent and production.3 The study takes up the two main issues dealt with in the literature cited above, i.e., asset pricing and output determination. The basic

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2. KIRMAN (1992) gives easy access to this literature while RAINEY'S (1982) survey covers older and more technical points.
3. The effects on land rent and land price are qualitatively the same if the interest rate is unaffected by the heterogeneity of beliefs. No attempt at modeling the interest rate is made in this paper.

setup follows NIEHANS (1994). We study an economy where individuals produce a commodity by means of a factor of production called land. A market operates on which land owners can lend their land to other producers. Land stands as an example for factors of production whose input levels can be varied by the producer. Dispersion of expectations is captured by modeling two types of individuals: optimists who over-predict and pessimists who under-predict their success.\(^4\)\(^5\) The present study confirms KAZEMI'S finding that dispersion of beliefs has ambiguous effects on prices. Here, however, the direction of the effect depends on the object of disagreement rather than on the reason for disagreement.

Furthermore, the models developed illustrate how land rent and production are determined when heterogeneity of expectations is combined with a possible disparity of optimists and pessimists in the population. Population expectations that, in their mean, are significantly biased away from the rational value have been documented in the empirical literature on the rational expectations hypothesis [see, e.g., FIGLEWSKI and WACHTEL (1981) and LOVELL (1986)]. KEYNES' (1936) notion of waves of optimistic and pessimistic sentiment captures the idea that average bias can change its direction over time.\(^6\) Evidence gathered by psychologists indicates that people—males in particular—tend to overestimate their own abilities [see FRIEZE et al. (1978)]. Thus, while expectations may be biased towards pessimism some of the time, more often they seem to be biased towards optimism.\(^7\) Independent of the direction of average bias it seems

\(^4\) NIEHANS' (1994) model of a land market captures differences in beliefs by modeling three types of individuals: optimists, pessimists and realists. He assumes that the fact that an agent's judgment about the future is deficient (i.e., he either is an optimist or a pessimist) implies that he obtains less output from a given input than a realist. NIEHANS' analysis then focuses on the welfare cost that occurs when deficiencies in the assessment of the future are associated with illusions about one's own entrepreneurial abilities. This association is not present in this setup. Here agents' productivity is no lower because they have deficient forecast abilities. Losses occur only because agents either over- or under-allocate resources due to biased foresight. Thus, in the present models realists are an irrelevant category and therefore omitted. The difference in setup makes the difference in outlook between NIEHANS' and my analysis. The present setup opens the way for an analysis of the effects of dispersion of expectations (i.e., by how much optimists and pessimist differ) and also of the effect of a possible average bias of expectations (i.e., a disparity of optimists and pessimists). A possible connection, however, between a person's ability of judgment and his productivity remains unexplored. Finally, the differing modeling strategies imply that the sense in which the two authors use the term welfare cost of optimism (or of pessimism in my version) is very different.

\(^5\) As McNees (1981) has shown with the example of macroeconomic forecasters it is easily possible for pessimists to turn into optimists and vice versa. The relevance of this study does not rest on the assumption that any given individual systematically under- or over-predicts future outcomes. The crucial point of departure is rather that at any given point in time there are over- and under-predictors.

\(^6\) KEYNES' sentence on the subject deserves to be quoted in full: "In abnormal times in particular, when the hypothesis of an indefinite continuance of the existing state of affairs is less plausible than usual even though there are no express grounds to anticipate a definite change, the market will be subject to waves of optimistic and pessimistic sentiment, which are unreasoning and yet in a sense legitimate where no solid basis exists for a reasonable calculation." p. 154.

\(^7\) WALDMAN (1994) suggests that this may be an optimal response of evolution to maladaptive preferences.
advisable to build models that allow investigation of its effects. Average bias in the present framework means that there is a disparity between optimists and pessimists. A last point this article addresses is the view that pessimism as such is socially harmful. This view is also associated with Keynes who was, however, mainly concerned with employment effects. The present setup looks at a situation where all factors of production are fully employed and hence focuses purely on allocative effects.

2. MODELS

The most concise way of formulating dispersion in expectations (or beliefs) is to assume that there are just two types of individuals, optimists and pessimists indexed by $o$ and $p$. The individuals produce a commodity with the variable factor of production called land ($L$). Everybody produces with the same production function

$$Q^o_f = L_o - bL^2_o$$
$$Q^p_f = L_p - bL^2_p$$

with $1/(2L^*) > b > 0$.\(^8\)

Production is subject to diminishing returns because there is a second factor of production called management skill of which every producer has a fixed amount. The superscript $ef$ indicates that this is the effective output produced by an individual. Given that the production functions are the same for all, a socially efficient land use demands that all use the same amount of land. This, in short, is the rational expectations equilibrium. All emerging allocations will be judged against this benchmark. A land market allows them to rent land from others who will be, in the present setup, those with different expectations.

Conceptually, the difference between optimists and pessimists can be captured in various ways. The terms optimism and pessimism are closely tied to the concept of subjective probabilities. This does not mean that a model with optimists and pessimists has necessarily be a stochastic model. As a matter of fact, none of the model variants presented in the main text are explicitly probabilistic. In our framework future rewards are fully determined by agents' present efforts; agents, however, are liable to make errors assessing the quantitative link between the present and the future. Consider the example of a «test of strength hammer» at a country fair. Here, it is simple to classify individuals into optimists and pessimists. Optimists are those who over-predict their abilities while pessimists under-predict their abilities. Similarly, producers can be classified according to their propensity to over- or under-predict their success. The appendix shows the equivalence of the approach used here with a stochastic formulation where probability assessments are an object of disagreement and misjudgment. The three model variants

\(^8\) $L^*$ is larger than any equilibrium input level considered in the following. Parameter restrictions similar to the one for (1) also apply to (2), (8), and (11).
in the main text will show land rent and output as a function of the size of individual errors and the overall composition of the population. The fraction of pessimists in the population will be used as a measure of composition. The three versions of the model illustrate how the effects of heterogeneity of expectations depend on the object of disagreement.

2.1 Errors Regarding the Intercept of the Marginal Product Curve

Optimists and pessimists both misperceive their productivity: optimists over-predict output at every level of input while pessimists under-predict output. We start by describing the situation of the individuals and the optimization problem. The first way to formalize individual expectational errors is the following

\[ Q_s = (1 - a) L_p - b L_p^2 \]
\[ Q^*_s = (1 + a) L_o - b L_o^2. \]  

(2)

The superscript \( s \) denotes that these are subjectively perceived production possibilities.\(^9\)

Here, \( a \) is a constant equal to or larger than zero that measures the individual error in expectations about productivity. A change in \( a \) shifts the intercept of the marginal product curve. Here, the assessment of the level of the intercept of the marginal productivity curve is the object of disagreement. The larger \( a \) the more optimists err on the upper side and the more pessimists err on the lower side. The two (subjective) income functions for the two types of individuals are

\[ Y_p = Q_p - (L_p - \bar{L}_p) r \]
\[ Y_o = Q_o - (L_o - \bar{L}_o) r. \]  

(3)

Here, \( \bar{L}_p \) and \( \bar{L}_o \) denote the land endowments of the two sorts of individuals and \( r \) is the rental price for land. It is assumed that individuals want to maximize their income. After deriving first order conditions we can write the two land demand functions:

\[ L_p = \frac{1 - r - a}{2b} \]
\[ L_o = \frac{1 - r + a}{2b}. \]  

(4)

From the demand functions we take the step to the equilibrium on the land market.

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9. As a reminder (1) describes effective production possibilities.
\[ \alpha L_p + (1 - \alpha)L_o = \bar{L}, \]  

where \( \alpha \) denotes the fraction of pessimists and \((1 - \alpha)\) the fraction of optimists and \( \bar{L} \) is the fixed land supply.\(^{10}\) The resulting equilibrium land rent is

\[ r = 1 - 2b\bar{L} + a(1 - 2\alpha). \]

The first two terms on the right hand side indicate the rational expectations land rent: the outcome when everybody has correct judgment (i.e., \( a = 0 \)). However, this level of land rent also results when producers make errors (i.e., with \( a > 0 \)) but \( \alpha = 0.5 \). The present setup thus supports the proposition regarding market equilibrium cited initially: any level of heterogeneity of beliefs produces the same outcome as under rational expectations as long as individual errors cancel out over the whole population. This confirmation, however, is restricted to the result concerning land rent. As the following passage will show, output is not invariant to the dispersion of beliefs.

The land rental in the present setup is lowest when the market consists only of extreme pessimists. This can also be seen in figure 1 where the numbers indicating the level of

Figure 1: Land Rent as a Function of the Size of the Individual Expectations Error (\( a \)) and the Fraction of Pessimists in the Population (\( \alpha \))

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10. Equation (5) follows from \( n_pL_p + n_oL_o = \bar{L}_{\text{total}} \) where \( n_p \) and \( n_o \) denote the numbers of pessimists and optimists. The \( \alpha \) in (5) then is \( n_p/(n_p + n_o) \) and \( \bar{L} \) is \( \bar{L}_{\text{total}}/(n_p + n_o) \), that is, average land supply.
the land rental are lowest in the north eastern corner. Intuition for this is straightforward: pessimists expect very low productivity and hence are unwilling to pay much for rented land. For the simulations underlying the figures the values $b = 0.25$ and $L = 1$ are used. These numbers lead to a benchmark land rent of 0.5. The maximum of $a = 0.15$ in the figure means an (absolute) error of 20% at the input level $L = 1$. All figures to follow are scaled in this same way.

Output is the weighted average of optimists’ and pessimists’ production. In order to derive it (6) is inserted into (4) to get the land uses of the two types of individuals and these, in turn, are inserted into (1). Per capita output then is

$$Q = \alpha Q^f + (1 - \alpha) Q^p,$$  \hfill (7)

which in this simplest case can be written as

$$Q = \bar{L} - b\bar{L}^2 - \frac{a^2}{b} (1 - \alpha) \alpha.$$  \hfill (7')

The first two terms on the right hand side indicate benchmark output while the third term shows the output loss due to expectational error. Output is clearly highest when there are no expectations errors (i.e., $\alpha = 0$). However, when there are expectations errors it would be socially desirable to have complete uniformity in opinions (i.e., $\alpha = 0$ or $\alpha = 1$). It does not matter whether all individuals are optimists or pessimists but it is clearly efficient for all individuals to be equal. In this case the market allocates the same amount of land to every individual just as in a situation where all have perfect knowledge. The worst case is when, with non-zero expectations errors, the two groups of optimists and pessimists are of equal size ($\alpha = 0.5$). Figure 2 shows output as a function of the two parameters $a$ and $\alpha$ (the benchmark level of output is 0.75 given the numerical values of $b = 0.25$ and $L = 1$). The arrow shows the fraction of pessimists that minimizes output at any given size of the individual forecast error. The starting point of the arrow is the rational expectations solution of the model. In figure 2 this arrow also indicates the direction of maximum decline in output in the $a/\alpha$-plane.

Hence, with a positive level of individual expectation error (i.e., $a > 0$) on average unbiased expectations (i.e., $\alpha = 0.5$) are not socially optimal. If we consider the possibility that, at levels of $a > 0$, the composition of optimists and pessimists is not fifty-fifty then output increases when the population is either biased toward optimism or toward pessimism. This example shows that it is wrong to believe that pessimism is necessarily welfare reducing. Furthermore, in the present case the production level is furthest away from the rational expectations solution when individual errors average out to zero.
2.2 Errors Regarding the Slope of the Marginal Product Curve

As in the previous section, we start by describing the situation of the individuals and the optimization problem. Here, the subjectively perceived production functions are

\[
\begin{align*}
Q_p &= L_p - (b + h)L_p^2 \\
Q_n &= L - (b - h)L_n^2
\end{align*}
\]

The parameter \( h \) indicates by how much individuals misperceive the slope of the marginal product curve. As with \( a \) in the previous section, \( h \) is equal to or larger than zero. Here then the assessment of the slope of the marginal productivity curve is the object of disagreement. The resulting land demand functions are

\[
L_p = \frac{1 - r}{2(b + h)}
\]
\[ L_\omega = \frac{1 - r}{2(b - h)} \]  

Using the equilibrium condition for the land market equilibrium land rent can be written as

\[ r = \frac{1 - 2bL + \frac{h}{b} (2Lh + 1 - 2\alpha)}{1 + \frac{h}{b} (1 - 2\alpha)}. \]  

Figure 3 shows land rent as a function of the two parameters \( h \) and \( \alpha \). This figure documents that, contrary to the previous setup, the asset price is not in general equal to the rational expectations equilibrium when individual errors average out to zero. This can be seen by reading the (increasing) numbers as we move up the solid vertical line. The lower end of this line again indicates land rent under rational expectations.

**Figure 3: Land Rent as a Function of the Size of the Individual Expectations Error (h) and the Fraction of Pessimists in the Population (\alpha)**

Output can be derived by inserting (10) into (9) which, in turn, is inserted into (1). Finally, the resulting \( Q^e_\omega \) and \( Q^e_{\theta} \) go into (7). Output emerges as a complicated expression of all

parameters introduced. Figure 4 shows per capita output as a function of the two parameters \( \alpha \) and \( h \). At any positive level of individual misperception, \( h \), output and hence welfare are lowest when, on average, there is a bias towards pessimism (i.e., when the fraction of pessimists is larger than 0.5). The arrow in figure 4 shows (starting from the rational expectations level of output) the direction of decline of output in the \( \alpha/h \)-plane. Let us consider disparity between optimists and pessimists against the background of positive levels of expectations errors. An increase in the fraction of pessimists in the population (i.e., an \( \alpha < 0.5 \)) is welfare decreasing as long as \( \alpha \) lies within the corridor bordered by the thin lines. Clearly, small to rather substantial majorities of pessimists are output decreasing. A large enough pessimist bias, however, leads to an output level higher than that in a situation of complete parity between optimists and pessimists. Again, it would be wrong to assert a general association of pessimism with lowered welfare.

Figure 4: Output as a Function of the Size of the Individual Expectations Error (\( h \)) and the Fraction of Pessimists in the Population (\( \alpha \))
2.3 Errors Regarding the Level of Output

In the present formulation of the problem misperception of the input-output relationship is modeled so that at any level of input expected output deviates from actual output by a constant fraction. Hence, both the intercept and the slope of the marginal product curve are subject to possible misperception:

\[
Q_p^* = (1 - d)(L_p - bL_o^2) \\
Q_o^* = (1 + d)(L_o - bL_o^2)
\]  

(11)

Here, \(d\) is the percentage misperception of output. After deriving the first order conditions for land use we reach the land demand functions:

\[
L_p = \frac{1 - r - d}{2(1 - d)b} \\
L_o = \frac{1 - r + d}{2(1 + d)b}
\]  

(12)

We proceed as in the previous setups in order to determine the market equilibrium. Equilibrium land rent is:

\[
r = \frac{(1 - 2bL_o)(1 - d^2)}{1 + d(1 - 2\alpha)}
\]

(13)

Figure 5 shows land rent as a function of the two parameters \(\alpha\) and \(d\). Again, land rent is not equal to the rational expectations value when individual errors cancel out on average. This can be seen by reading the (decreasing) numbers as we move up the solid vertical line. Figure 6 shows output as a function of the two parameters \(\alpha\) and \(d\). Output is maximized when either there are no expectations errors (i.e., \(d = 0\)), or when all individuals make the same expectations error (\(\alpha = 1\) or \(\alpha = 0\)). At any positive level of individual misperception, \(d\), output and hence welfare are lowest when there is an aggregate bias towards optimism (i.e., when the fraction of pessimists is less than 0.5). The arrow in figure 6 (starting from the rational expectations level of output) shows the direction of decline of output in the \(\alpha/d\)-plane. Let us consider biased average expectations given non-zero individual expectations errors. An increase in the fraction of optimists in the population (i.e., an \(\alpha < 0.5\)) is welfare decreasing as long as \(\alpha\) lies within the corridor bordered by the thin lines. Clearly, in the case at hand a small majority of optimists over pessimists is output decreasing while large majorities of optimists are output increasing. Contrary to the previous setup, here optimism (not pessimism) is
potentially damaging. But a large enough bias towards optimism can be socially beneficial.

As a last point it should be noted that results similar to the ones discussed here emerge when all producers err in the same direction but by different degrees. Hence, with everybody an optimist (or pessimist) but with different amounts of individual over-prediction (or under-prediction) land rent and output also deviate from their rational expectations values.

Figure 5: Land Rent as a Function of the Size of the Individual Expectations Error (d) and the Fraction of Pessimists in the Population (α)
3. SUMMARY AND CONCLUSIONS

The most important result is that heterogeneity of expectations matters. Output clearly falls with rising expectations errors. This occurs even – and strongly – when there are equal numbers of over- and under-predictors in the economy. This shatters the belief which is still widely held that market outcomes equal the predictions from rational expectations models as long as individual idiosyncrasies cancel out on average. This study shows that whether, at a given level of individual misjudgment, it is socially worse to have a small to moderate majority of optimists or pessimists depends on the object of disagreement and misjudgment. In all cases studied the misjudgment concerns the input-output relationship. However, it makes a difference whether producers typically misjudge the intercept or the slope, or both, of the marginal product curve. Hence, a general assertion of the kind that an economy with a majority of pessimists (i.e., under-predictors) is worse off than an economy with a majority of optimists (i.e., over-predictors) or the reverse is untenable. Interestingly, large disparities between over- and under-predictors usually have different effects than small disparities. At the limit,
with everybody an optimist (or pessimist) output is again at its maximum. What counts therefore is not the individual error as such but the coexistence of people with different beliefs. Expectations errors, in the models of this paper, do not induce welfare losses as long as there is conformity in the population.

Land rent, unlike output, can rise with rising individual expectations errors given parity between over- and under-predictors. The three cases studied show that land rent can also remain unchanged or fall with a rising expectations error. Generally, land rent is lowest when the population consists only of pessimists. However, output is at its maximum under a variety of circumstances. Hence, the level of land rent and the level of output and welfare are not connected in clear and simple ways.

In summary, both the prices of productive assets of an economy and the output level depend on the ability of producers to make quantitative assessments. Improving this ability through schooling is likely to produce welfare gains. As this study suggests similarity of assessments can mitigate the losses due to wrong assessments. Hence, conformity can be a social virtue and individualism can be a burden.

APPENDIX

This appendix shows the basic equivalence of the setup used in this study and the state space approach. Here, I formalize the problem described under 2.1 in a probabilistic way.\textsuperscript{11} The individuals in this model variant assign different subjective probabilities to two possible states of the world. Individuals and states of the world are described in the following. There are two states of the world: a more productive state (m) and a less productive state (l). In the l-state of the world output is just

\[
Q_l = (1 - g) L - b L^2
\]

while in the m-state it is

\[
Q_m = (1 + g) L - b L^2,  \tag{A1}
\]

where \( g \) is a constant equal to or larger than zero. The two sorts of individuals assign different subjective probabilities to the two possible states:

\[
SPR_p \text{, } l\text{-state } = \gamma, \text{ } SPR_p \text{, } m\text{-state } = 1 - \gamma
\]

\[
SPR_o \text{, } l\text{-state } = 1 - \gamma, \text{ } SPR_o \text{, } m\text{-state } = \gamma  \tag{A2}
\]

Here, \( SPR \) stands for subjective probability and \( \gamma > 0.5 \). This means that pessimists assign a higher probability to the less productive state while optimists assign a higher prob-

\textsuperscript{11} This formulation follows HEY (1984).
ability to the more productive state of the world. Subjectively expected output for the two types of individuals can now be written as

\[ Q_p' = [1 - (2\gamma - 1)g]L_p - bL_p^2 \]
\[ Q_o' = [1 + (2\gamma - 1)g]L_o - bL_o^2 \]  \hspace{1cm} (A3)

These two equations show the equivalence indicated above: the term \((2\gamma - 1)g\) in (A3) plays the same role as \(a\) in (2). In fact, with \(\gamma = 1\) (i.e., individuals believe fully in the occurrence of one outcome) (A3) becomes

\[ Q_p = (1 - g)L_p - bL_p^2 \]
\[ Q_o = (1 + g)L_o - bL_o^2 \]  \hspace{1cm} (A3')

Hence, \(g\) in (A3') plays the same role as \(a\) in (2) and the analysis of the two models becomes interchangeable.

REFERENCES


12. For a welfare analysis, it would be straightforward to assume that objectively the two states have equal probabilities (i.e., \(P_{\text{r-state}} = P_{\text{m-state}} = 0.5\)).

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SUMMARY

This paper shows that heterogeneity of expectations matters. The individual differences studied concern the assessment of the input-output relationship. Output clearly declines with rising expectations errors. This occurs even – and strongly – when there are equal numbers of over- and under-predictors in the economy. This shatters the belief which is still widely held that market outcomes equal the predictions from rational expectations
models as long as individual idiosyncrasies cancel out on average. Depending on the precise object of disagreement, it can be worse to have a majority of pessimists or a majority of optimists. The study suggests that similarity of assessments can mitigate the losses due to wrong assessments. Hence, conformity can be a social virtue and individualism can be a burden.

RESUME

Cette étude concerne l’hétérogénéité des expectatives. Nous étudions les effets des différences individuelles concernant l’estimation de la productivité. Dans une population de producteurs, il y a les optimistes qui surestiment leur productivité et les pessimistes qui la sousestiment. L’analyse montre que le produit total d’une économie hétérogène est plus bas comparé au produit d’une économie formée d’agents rationnels. L’article donne aussi des réponses à la question s’il est pire d’avoir une majorité de pessimistes ou une majorité d’optimistes dans une économie.